Subject to the assumptions that the flow is inviscid, non-rotating, linear and hydrostatic, the Taylor-Goldstein equation governs the behaviour of the Fourier transform of the vertical velocity perturbation $v$ associated with the waves:

$$\hat{v}(\mathbf{k}, \mathbf{\omega}) = \frac{1}{\sqrt{2\pi} \sqrt{\Delta^2 + k^2}} \int \left( \frac{\Delta^2 + k^2}{\Delta^2 + k^2} \right)^{1/4} e^{-i \mathbf{k} \cdot \mathbf{r}} e^{-i \mathbf{\omega} \cdot \mathbf{t}} \, d^3 \mathbf{r} \, d^3 \mathbf{t}$$

where $U$ and $V$ are the velocity and Brunt-Väisälä frequency of the incoming flow and $(\partial / \partial z)$ is the horizontal wavenumber vector of the waves. If the flow varies slowly with height, the solution to this equation takes the form:

$$w(Z) = \int_{-\infty}^{\infty} \left( \frac{U}{U} \frac{U}{U} + \frac{V}{V} \frac{V}{V} \right) e^{-i \mathbf{k} \cdot \mathbf{r}} e^{-i \mathbf{\omega} \cdot \mathbf{t}} \, d\mathbf{r} \, d\mathbf{t}$$

where $W = U \cos \theta - V \sin \theta + \omega \cdot \mathbf{r}$.

For both wind profiles considered here, expressions for $\Delta \omega / \Delta \lambda$ and $\Delta \Delta \omega / \Delta \lambda$ can be obtained in closed analytical form. Although the examples presented above only consider wind profiles where the wind angle varies monotonically, non-monotonic variations, with more than one critical level, can be incorporated by splitting the atmosphere into multiple layers. The momentum flux can be related to its value at the bottom of the layer (here denoted by $\omega = 0$) using:

$$M_{\text{base}} = C M_{\text{base}}(z = 0) + C M_{\text{base}}(z = 0)$$

where the coefficients $C_{\text{base}}$ only depend on the atmospheric parameters and aspect ratio $\gamma$.

Conclusions

The WKB calculations presented here incorporate, in a self-consistent way, wind profile effects on the drag and momentum flux from hydrostatic mountain waves containing a full spectrum of scales. This is particularly important for correctly representing the variation of the momentum flux with height for winds with directional shear over 3D mountains due to critical level filtering, an effect that is potentially very important for non-breaking mountain waves, and is treated here in the linear-wave approximation. The present work extends this framework to mountains with an elliptical cross-section (used as building blocks in drag parametrizations) and wind profiles with arbitrary turning with height. More details are available in Teixeira & Yu [4].