Department of Meteorology

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Drag produced by trapped lee waves and upward propagating mountain waves in directional shear flow

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Summary

We calculate the gravity wave drag exerted on an axisymmetric mountain by a two-layer stratified atmosphere where the wind varies linearly in the lower layer and is constant aloft. Unidirectional flow with positive or negative shear, and flow with directional shear of various types are considered. The drag oscillates with the thickness of the constant-shear layer and the Richardson number within it (*Ri*), generally decreasing for low *Ri* and strong non-hydrostatic effects. Critical-level absorption, which increases with the angle spanned by the wind in the constant-shear layer, shields the surface from reflected waves, keeping the drag closer to its hydrostatic limit. A substantial drag fraction may be produced by trapped lee waves, particularly when the flow is strongly non-hydrostatic, the lower layer is thick and *Ri* is relatively high. In directionally sheared flow with $Ri = O(1)$, the drag may be misaligned with the surface wind in a direction opposite to the shear, a behaviour totally due to non-trapped waves. The trapped lee-wave drag, which acts on the atmosphere at low levels, may therefore be misaligned with the drag produced by vertically propagating waves, which acts higher in the atmosphere.

1. Introduction

Teixeira et al. [1] showed recently that, in non-hydrostatic flow over relatively narrow mountains, the drag produced by mountain waves receives contributions from waves that propagate vertically and waves that are trapped near the surface. This complicates the drag evaluation for idealized cases relevant for drag parametrization development, because the trapped lee waves arise due to singularities in the wave spectrum, corresponding to discrete modes, at least in flow over 2D orography. Most drag calculations due to trapped lee waves have focused on very simple piecewise-constant atmospheric profiles over 2D orography, because that allows the drag to be obtained analytically. However, one of the most common wave trapping mechanisms is vertical wind shear. Keller [2] and Shutts [3], for example, investigated mountain waves in idealized unidirectional and directional shear flows, respectively, but they did not focus on the drag. On the other hand the study of Teixeira et al. [4], which can be viewed as a precursor to the present one, considered two-layer directional shear flow, but was restricted to hydrostatic conditions, where no trapped lee waves can exist. Here we extend this approach to non-hydrostatic conditions, and address the drag produced by the highly complex flow configuration associated with trapped lee waves existing in directional wind shear over an axisymmetric mountain.

2. Theoretical model

We use a semi-analytical model based on the Taylor-Goldstein equation:

$$\hat{w}'' + \left\{ \frac{N^2 k_{12}^2}{(\mathbf{U}\cdot\mathbf{k})^2} - \frac{\mathbf{U}''\cdot\mathbf{k}}{\mathbf{U}\cdot\mathbf{k}} - k_{12}^2 \right\} \hat{w} = 0, \qquad (1)$$

where $\hat{w}$ is the Fourier transform of the vertical velocity perturbation associated with the wave, (U,V) is the wind velocity and N is the Brunt-Vaisala frequency of the incoming flow, $\mathbf{k}$ is the horizontal wavenumber of the waves, $|\mathbf{k}|=k_{12}$, and the primes denote differentiation with respect to z. The vertical velocity perturbation w must satisfy the free-slip boundary condition: $w(z=0) = \mathbf{U}_1(z=0)\cdot\nabla_H h(\mathbf{x})$ (2)
where $\mathbf{U}_1$ is the incoming wind velocity in the lower layer and h is the surface elevation. w must also either decay or satisfy a radiation boundary condition as $z\to\infty$. The wind profile is assumed to take the form:

$$\mathbf{U}(z) = \begin{cases} (U_0 + \alpha_1 z, \alpha_2 z) & \text{for } 0 < z < H, \\ (U_0 + \alpha_1 H, \alpha_2 H) & \text{for } z > H, \end{cases} \qquad (3)$$

where H is the depth of the lower layer where the shear is constant $\alpha=(\alpha_1, \alpha_2)$. The solution to (1) in each layer is:

$$\hat{w} = \begin{cases} A\hat{w}^{\uparrow} + B\hat{w}^{\downarrow} & \text{for } 0 < z < H, \\ C\exp(imz) & \text{for } z > H, \end{cases} \qquad (4)$$

where A, B and C are coefficients, and m is the vertical wavenumber in the upper layer. $\hat{w}\uparrow$ and $\hat{w}\downarrow$ correspond to waves whose energies propagate upward and downward, respectively.

Solutions for $\hat{w}\uparrow$ and $\hat{w}\downarrow$ in the lower layer can be expressed in terms of modified Bessel functions, taking different forms above and below critical levels, for which $\mathbf{U}\cdot\mathbf{k}=0$. The ultimate aim here is to calculate the drag:

$$\mathbf{D} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(z=0)\nabla_H h(x,y)\,\mathrm{d}x\,\mathrm{d}y \qquad (5)$$

where p is the pressure perturbation, but the flow field is also addressed. Figure 1 illustrates the capabilities of the model in reproducing the w field from numerical simulations by Broutman et al. [5].

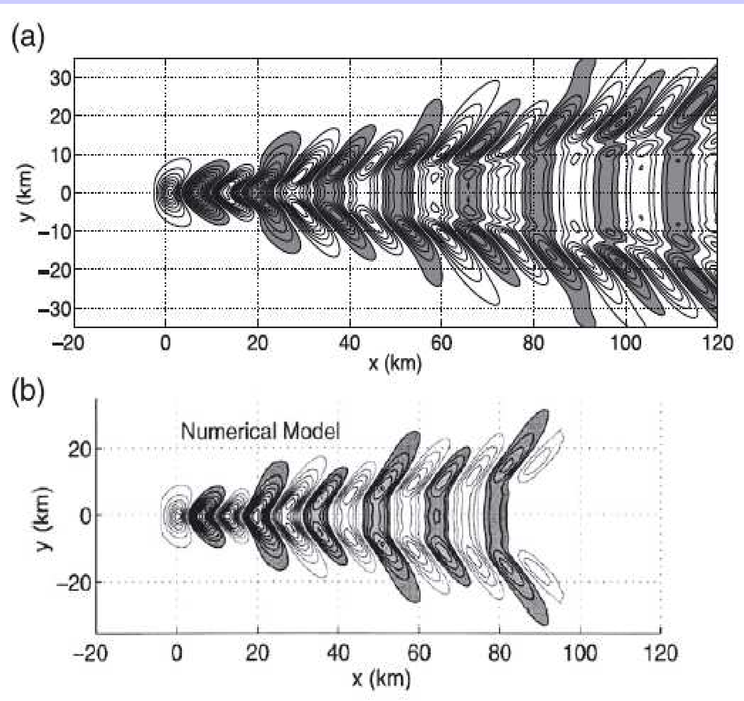

Figure 1. Density-scaled vertical velocity at a height z=2.5km. Shaded contours: positive values. (a) Present model, (b) Broutman et al. [5].

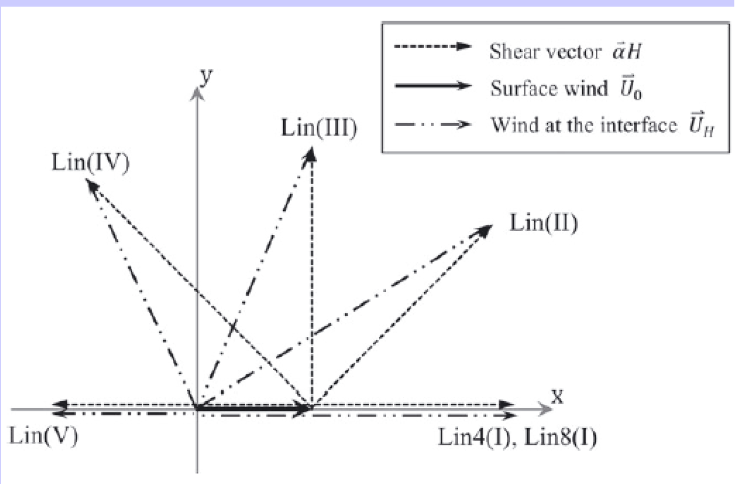

Figure 2. Schematic diagram showing wind profiles with different shear angles. Lin4(I) and Lin(V) have unidirectional shear.

Figure 2 shows the wind profiles to be considered. Results will be presented for Lin4(I), Lin(II), Lin(III), Lin(IV) and Lin (V). Since the flow is non-hydrostatic, the drag depends on the shape of the orography, which is assumed to take the form:

$$h(x,y) = \frac{h_0}{\{1+(x/a)^2+(y/a)^2\}^{3/2}} \qquad (6)$$

3. Results (unidirectional shear)

The drag normalized by its value in the absence of shear is a function of 4 dimensionless parameters: α_2/α_1, quantifying the direction of shear, $|\mathbf{U}(z{=}H)|/U_0$, quantifying the thickness of the shear layer, Ri, quantifying the shear intensity, and $\hat{a}=Na/U_0$, quantifying non-hydrostatic effects. $|\mathbf{U}(z{=}H)|/U_0=4$ will be assumed throughout, for illustrative purposes.

Consider first the case of unidirectional shear (α_2=0) (i.e. wind profiles Lin4(I) and Lin(V)). Figure 3 shows the normalized drag, and its fraction associated with trapped lee waves for Lin4(I), a wind profile with forward shear. The drag oscillates with Ri for nearly hydrostatic flow, slightly exceeding 1 at low Ri, but becomes lower, and more dominated by trapped lee wave drag as $\hat{a}$ decreases (i.e. non-hydrostatic effects intensify). At low $\hat{a}$, both the total drag and its trapped lee wave fraction decrease with Ri. Figure 4 shows the drag for a wind profile with backward shear, Lin(V). The drag varies more smoothly with Ri, generally decreasing with Ri, but attaining a maximum at low $\hat{a}$, and decreasing less with $\hat{a}$ at low Ri. The drag behaviour in Figure 3 can be attributed to vertical wave reflection and interference at the shear discontinuity existing at $z=H$ and total reflection of waves that become evanescent in the shear layer, and that in Figure 4 essentially to critical level wave absorption.

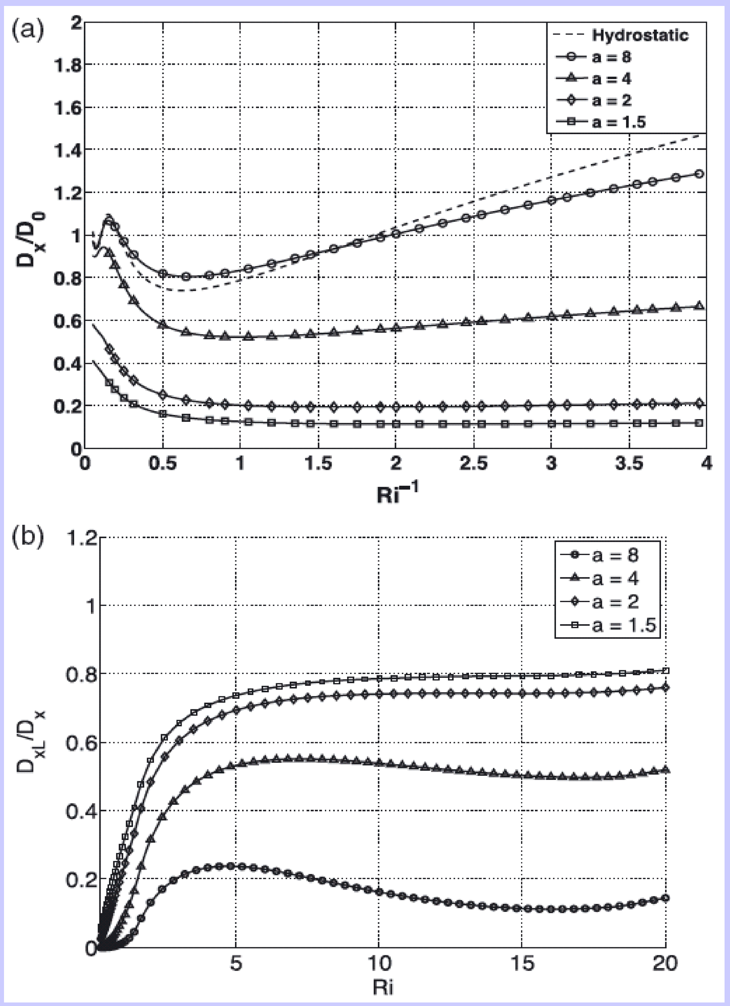

Figure 3. (a) Normalized total drag as function of Ri^{-1} and (b) its fraction associated with trapped lee waves as function of Ri, for wind profile Lin4(I).

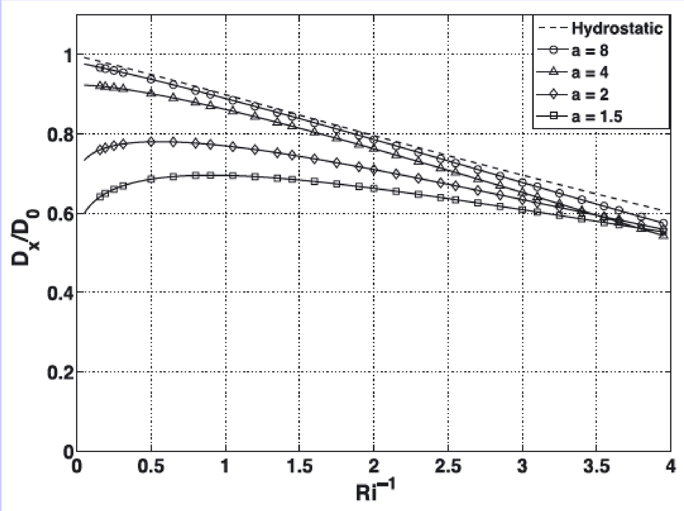

Figure 4. As Figure 3(a), but for wind profile Lin(V).

3. Results (directional shear)

Figure 5 shows the drag for wind profiles Lin(II), Lin (III) and Lin(IV). This corresponds to situations where the shear is oriented at angles of 45°, 90° and 135° to the surface wind, leading to directional wind shear. The drag behaves in an intermediate way relative to forward and backward shear, approaching the former for small angles (e.g. Lin(II)) and the latter for large angles (e.g. Lin(IV)). The drag is misaligned with the wind, in particular at low Ri D_y is negative despite the fact that the wind always turns counter-clockwise, with V>0 near the surface. This can only be attributed to wave reflection. It turns out that this is exclusively due to vertically propagating waves, as the trapped lee waves (and the associated drag) are always aligned towards the direction of the shear vector (counter-clockwise in this case). This means that the trapped lee wave drag (which acts at lower levels and typically is not represented in drag parametrizations) may be strongly misaligned with the drag associated with vertically propagating waves (which acts at much higher levels in the atmosphere). More details can be found in Yu & Teixeira [6].

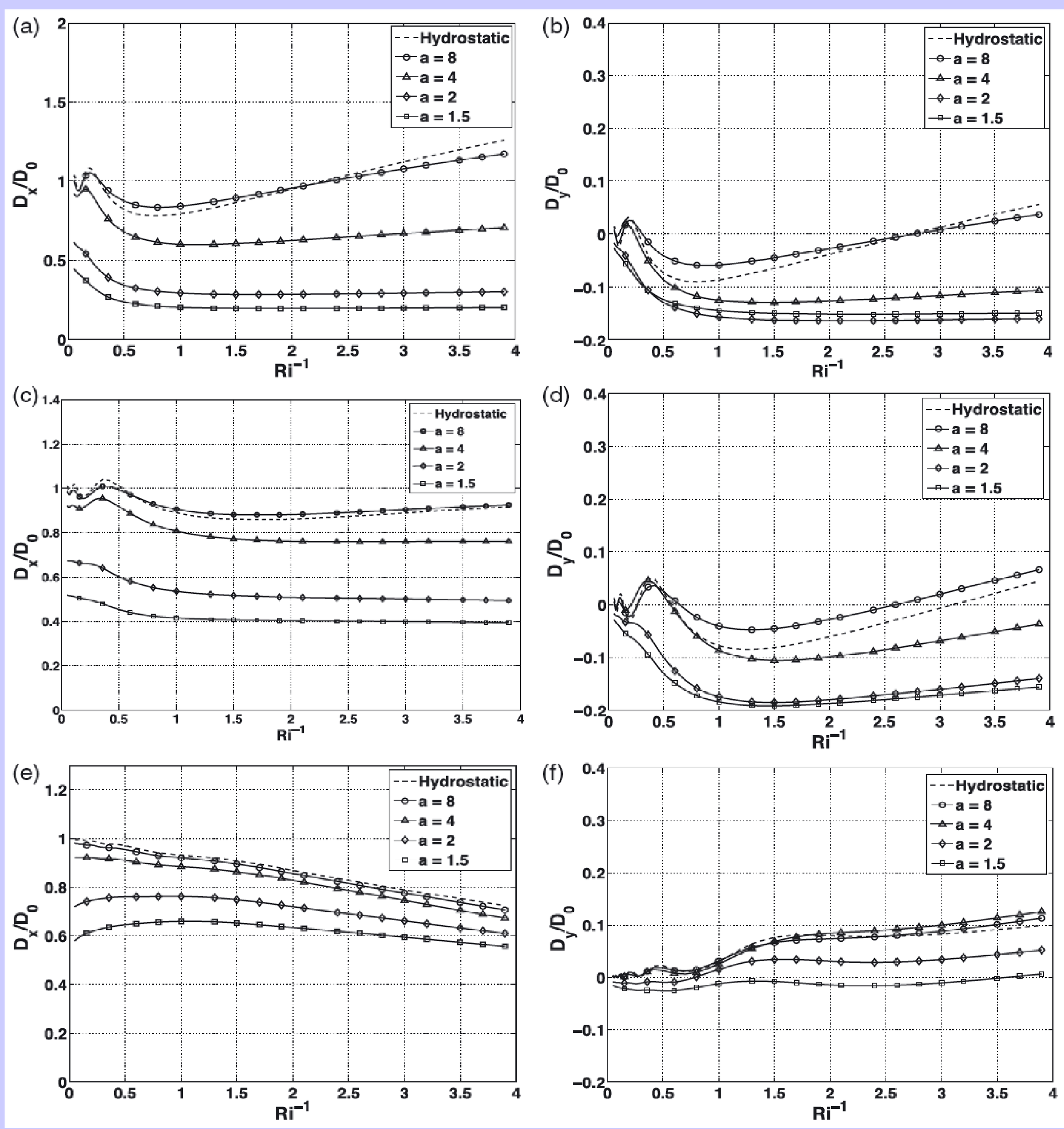

Figure 5. Normalized drag as a function of Ri^{-1}. (a) D_x/D_0 and (b) D_y/D_0 for Lin(II), (c) D_x/D_0 and (d) D_y/D_0 for Lin(III), (e) D_x/D_0 and (f) D_y/D_0 for Lin(IV).

Conclusions

The behaviour of the drag in the two-layer atmosphere addressed here depends on partial wave reflection at the shear discontinuity existing at the top of the lower layer, total reflection of waves that become evanescent due to non-hydrostatic effects as wind speed increases with height, and wave absorption by critical levels. These wave reflections lead to drag enhancement or weakening, while critical levels attenuate the effect of these reflections by absorbing the waves on their way up or on their return down to the surface. In directional shear flow, this leads to a trapped lee wave drag that may be substantially misaligned with the drag associated with waves that propagate vertically in the upper layer.

References

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