Stratification effects on lee waves trapped on the boundary layer inversion

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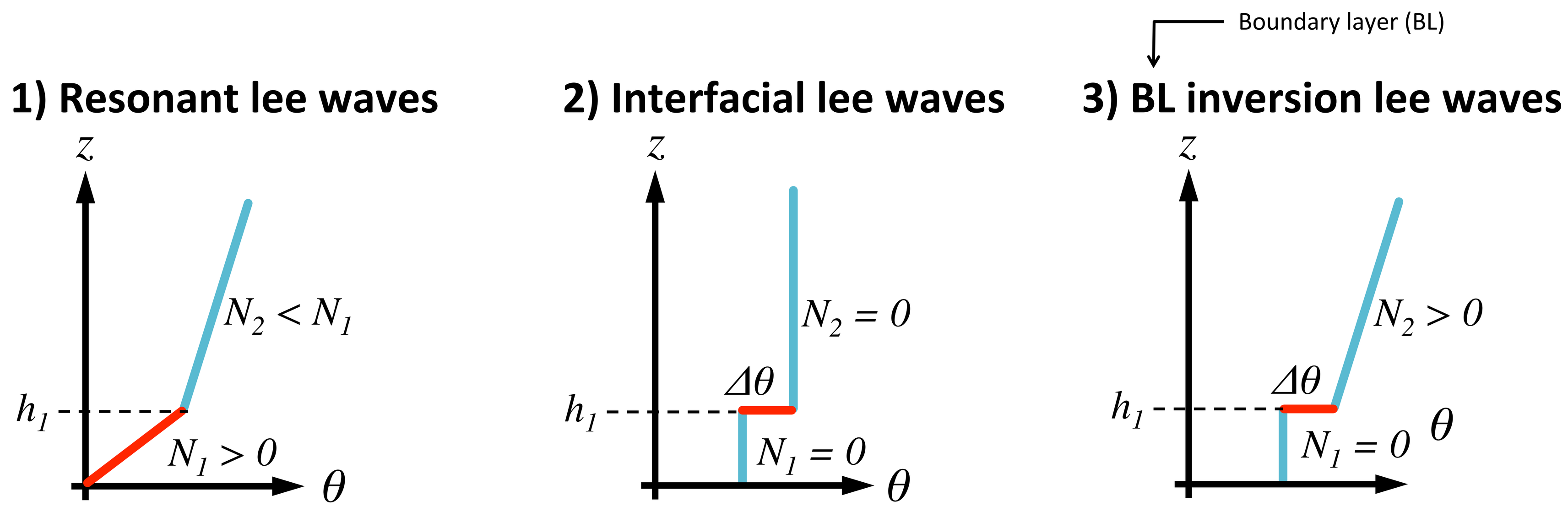
Introduction

- **Trapped lee waves** are **horizontally propagating waves** featuring **multiple wave crests** and can extend over several hundreds of kilometers.
- The wave **updraft regions** are visible in satellite images as **cloud stripe pattern** if sufficient moisture is available
- In flow with **uniform background wind speed** U, trapped lee waves are supported by different thermal structures (the wave trapping region is indicated in red):

1) Resonant lee waves | **2) Interfacial lee waves** | **3) BL inversion lee waves**



Well known, lots of literature. Relatively unexplored, little literature available.

Aims:

- **Validate** the **linear theory** of interfacial and BL lee waves **with observations** (Turner 1972, Vosper 2004)
- Study the **dependence** of the lee wavelength on stratification in the free atmosphere
- Discuss the **impact** of a continuously **stratified free atmosphere** on the applicability of **hydraulic analogies** for a quantitative description of layered **atmospheric flows**

Linear theory

We are interested in:

$\lambda = f(U, \Delta\theta, N_2, N_1 = 0, h_1)$

Wavelength of the trapped disturbance as a function of the wave parameters

Frequency dispersion relationship relates the phase speed c_p of waves with the wavenumber k

Frequency dispersion relationships (for stationary interfacial lee waves):

(Scorer 1949, Taylor 1972, Vosper 2004), $g' = g \cdot \Delta\theta/\theta_0$

In the focus of our investigations

Resonant lee waves

$$\coth(i m_1 h_1) = -\frac{l_2^2 - k^2}{l_1^2 - k^2}$$

Interfacial lee waves

$$U^2 = \frac{g'}{k\coth(kh_1) + k}$$

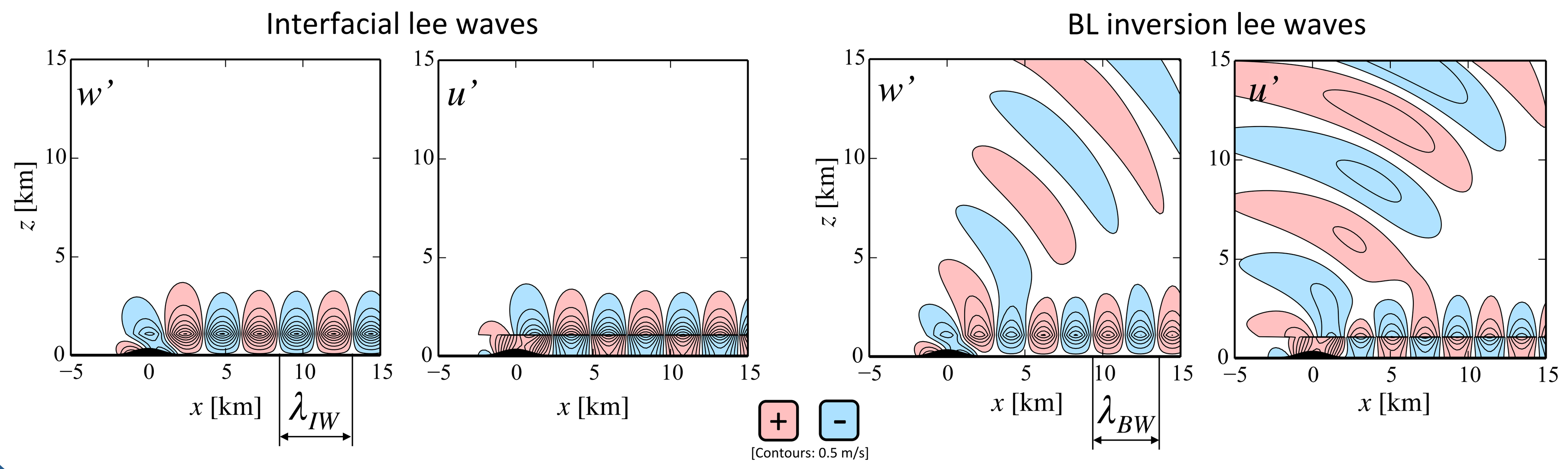
$$\lambda_{IW} = 2\pi/k$$

BL inversion lee waves

$$U^2 = \frac{g'}{k\coth(kh_1) + \sqrt{k^2 - \left(\frac{N_2}{U}\right)^2}}$$

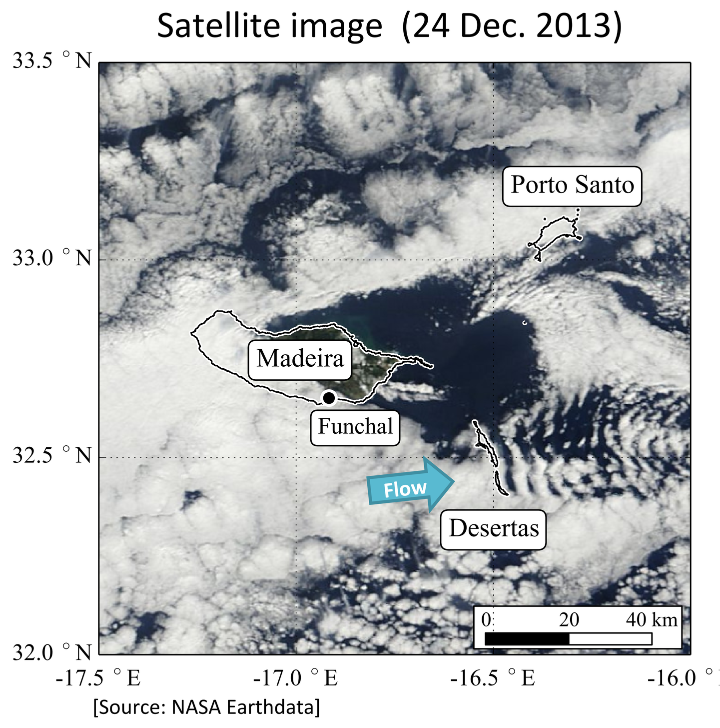
$$\lambda_{BW} = 2\pi/k$$

Vertical wave structure



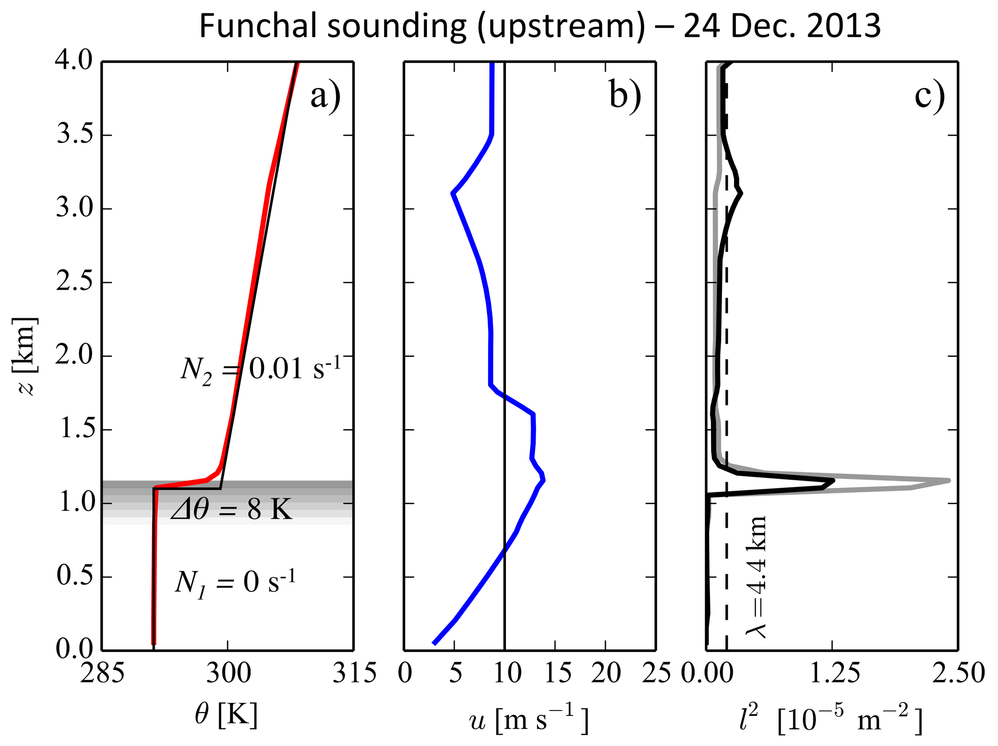
Observations – Madeira lee wave event

Satellite image (24 Dec. 2013)



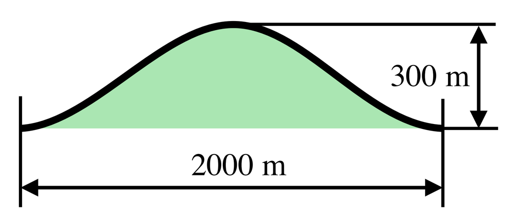
[Source: NASA Earthdata]

Funchal sounding (upstream) – 24 Dec. 2013



- **Why is this case interesting?**
  Upstream winds are relatively undisturbed compared to other mountain regions.
- **What do we use observations for?**
  We use the available data to validate the linear models of the previous section and apply representative atmospheric values to the FDRs.
- **Are there other regions where similar waves occur?**
  Yes, a similar boundary layer structure can be frequently observed over surrounding plains in mountain areas.

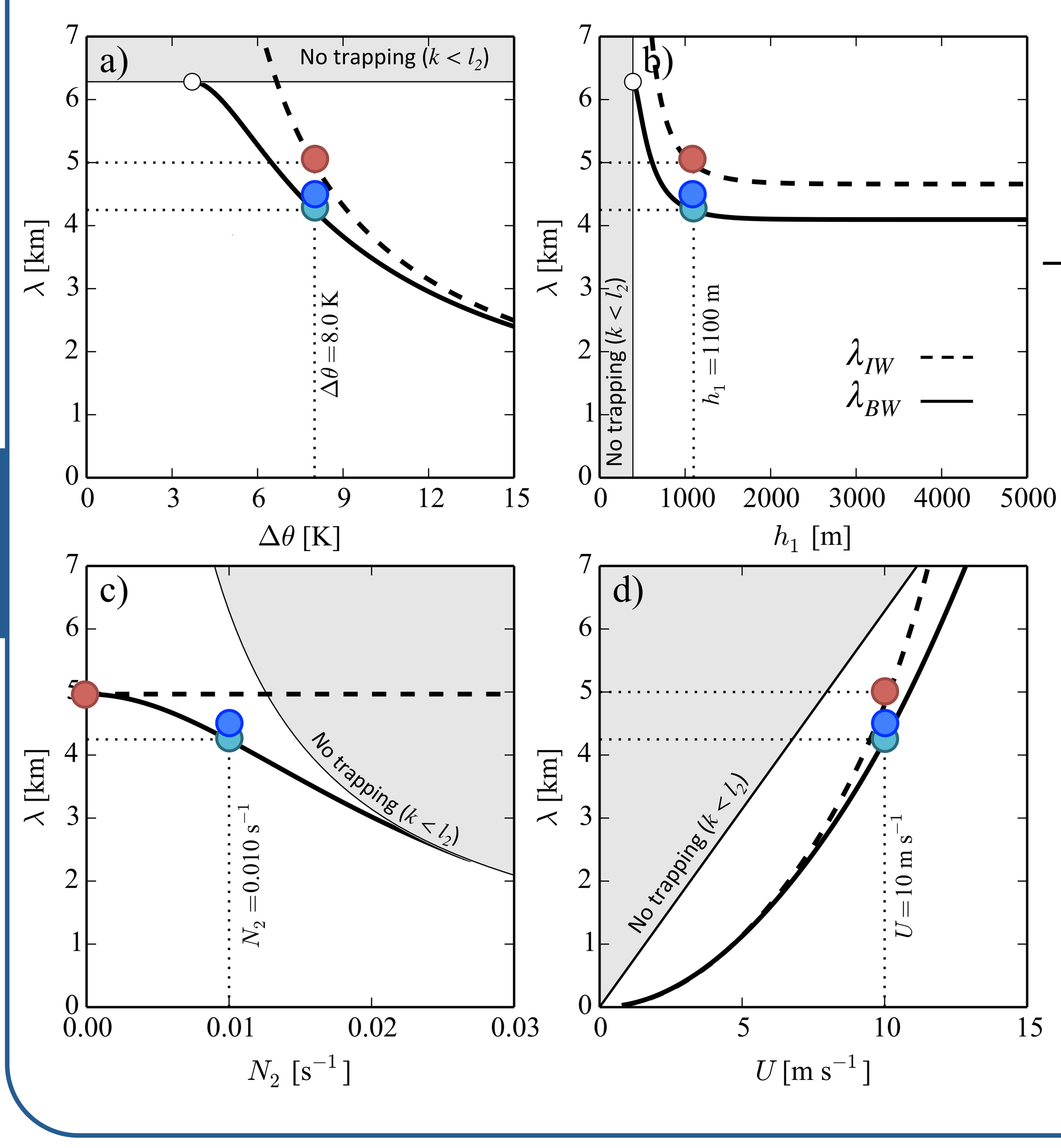
Dimensions of Desertas ridge:



Observed wavelength:

$\lambda_{OBS} = 4.4$ km

Linear theory – Wavelength results



Wavelength results	λ
Internal Interface wave: ($N_1 = 0$ s^{-1})	5.0 km
BL inversion wave: ($N_2 = 0.01$ s^{-1})	4.2 km
Observations:	4.4 km

Some remarks

- Linear theory gives accurate wavelength estimations
- The behaviour of λ_{IW} and λ_{BW} is generally similar
- Stratification decreases the wavelength (cf. dashed and solid lines).
- λ_{IW} and λ_{BW} become independent of the layer depth beyond a certain value of h_1.
- Critical values for wave trapping exist for $\Delta\theta$ and h_1 (white dots in (a) and (b))

How does the stably stratified free atmosphere affect the wavelength of a trapped lee wave?

Short answer: the wavelength generally decreases if $N_2 > 0$

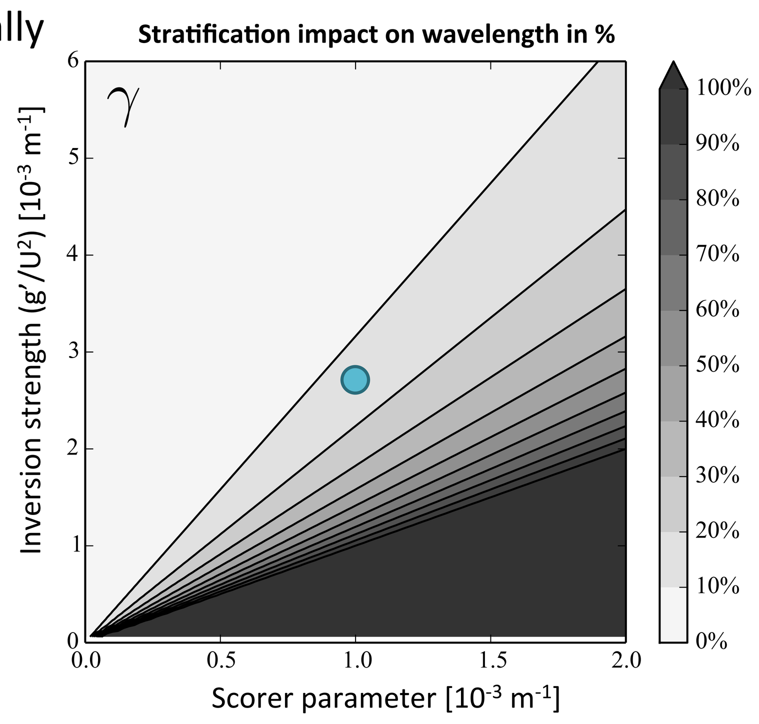
Long answer:
The wavelength impact can be estimated analytically

Relative difference between λ_{IW} and λ_{BW}

$$\gamma = \frac{N_2^2}{2g'} \Big/ \frac{g'}{2U^2} = \left(\frac{l_2}{\frac{g'}{U^2}}\right)^2$$

Valid for short waves ($kh_1 >> 1$)

Stratification effects are important when:
- winds are strong
- inversions are weak
- (trivially) stratification is strong



Desertas lee wave: increase of λ by a factor of 14 % due to stratification

Can hydraulic analogies be inappropriate if the atmosphere above an inversion is stably stratified?

Short answer: Yes, because the underlying long-wave approximation can be invalid if $N_2 > 0$

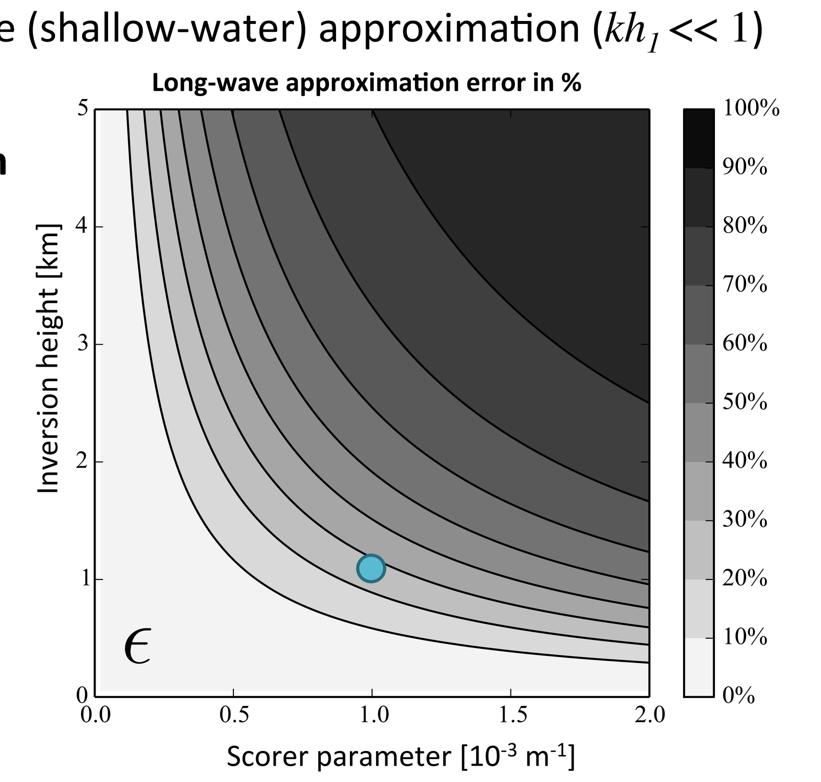
Long answer:
- Wave modes with $k > l_2$ can propagate through the inversion into the free atmosphere
- Only relatively short wavelength modes can be trapped on the interface
- However, hydraulic theory relies on the long-wave (shallow-water) approximation ($kh_1 << 1$)

Long-wave criterion $kh_1 << 1$ — incompatible when $N_2 > 0$ — **Wave-trapping criterion** $k > l_2$

Emerging error in long-wave approximation:

$$\epsilon = 1 - (l_2 h_1) \cdot \coth(l_2 h_1)$$

Hydraulic theory can be inappropriate if $N_2 > 0$
(because the long-wave (shallow-water) approximation is not valid anymore)



Desertas lee wave: $\epsilon \approx 27$ %

Summary

- Linear theory accurately predicts the wavelength of interfacial waves
- Stratification impact on interfacial trapped lee waves :
  1) The wavelength of the trapped disturbance decreases
  2) Only short-wavelength modes can be trapped on the interface
  3) For typical atmospheric values of Scorer parameter and inversion height, the shallow-water approximation is not valid anymore

Outlook

- Consider wave amplitude
- Validate with numerical simulations
- Include more observations

References
- Scorer, R. (1949). Theory of waves in the lee of mountains. Q. J. R. Meteorol. Soc. 75, 41–56
- Turner, J. S. (1973). Buoyancy effects in fluids (Cambridge University Press)
- Vosper, S. B. (2004). Inversion effects on mountain lee waves. Q. J. R. Meteorol. Soc. 130, 1723–1748

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